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Assessment of Analytic Methods for the Prediction of Supersonic Flow over Bodies

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Trends toward the automation of the design process for airplanes and missiles accentuate the need for analytic techniques for the prediction of aerodynamic characteristics. A number of computer codes have been developed or are under development that show promise of significantly improving the estimation of aerodynamic characteristics for arbitrarily-shaped bodies at supersonic speeds. The programs considered range in complexity from a simple linearized solution employing slender body theory to an exact finite-difference solution of the Euler equations. The results from five computer codes are compared with experimental data to determine the accuracy, range of applicability, ease of use, and computer time and cost of the program. The results provide a useful guide for selecting the appropriate method for treating bodies at the various levels of an automated design process. However, it is clear that the need still exists for a code that can account for the predominant nonlinear effects, operate from an easily-generated and manipulated geometry data set of the surface panel type, and execute in a reasonable time.

Nomenclature

C_p	= pressure coefficient, $(p-p_\infty)/q_\infty$
l	= body length
M	= Mach number
M_{local}	= local Mach number
p	= local static pressure
p_∞	= freestream static pressure
q_∞	= freestream dynamic pressure
X, Y, Z	= Cartesian coordinates
x	= longitudinal distance from model nose
α	= angle of attack, deg
β	= angle of sideslip, deg
θ	= angular coordinate, 0 deg on top side of model

Introduction

THE need for more accurate, more general, less restrictive, and faster computerized techniques for predicting configuration aerodynamic characteristics is accentuated by the trend toward automating the aircraft design process. To meet this need in the area of supersonic aerodynamics, several computer codes are under development that will satisfy at least some needs.

During the past several years, a considerable amount of progress has been made in the development of five such codes, ranging in complexity from a simple slender-body-theory linearized solution to a finite-difference Euler-equation solution. An assessment of the present situation regarding these codes is presented in this paper. The five specific codes are referred to as follows: general slender body,[1] Woodward triplet,[2] PAN AIR pilot,[3] modified linear theory,[4] and shock-fitting finite-difference.[5]

Present plans indicate that eventually each of these codes will be capable of performing the aerodynamic analysis of complete aircraft configurations (wind-body-tail configurations at a minimum); however, at present the codes are at various stages development (currently, only PAN AIR can handle completely general configuration geometries). This assessment is based on one common capability—the aerodynamic evaluation of arbitrarily-shaped fuselage-type bodies. Code results are evaluated by making comparisons with experimental pressure data which are part of an extensive experimental data base[6-9] for supersonic flow about analytically definable bodies.

The purpose of this paper is to assess the present situation regarding the development of five new supersonic computer codes. The assessment will consist of comparing computed pressures with experimental pressures for two body geometries at a variety of flow conditions. For each code, future plans for enhancements and implementations will also be presented.

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Prediction Methods

The following sections present the five prediction methods to be compared with experimental data. Computer program characteristics are summarized in Fig. 1.

General Slender Body

Of the five computer codes considered, the general slender body code is by far the simplest. In order to analyze the flow about a slender body of arbitrary cross-sectional shape, only two solution elements are needed. The first element is the solution for a body of revolution having the same cross-sectional area distribution as the actual slender body; the second element is a two-dimensional incompressible crossflow solution that is obtained by Theodorsen's method. The particular code used in this study was originally assembled by Bonner[1] and is being incorporated into a supersonic aerodynamic design and analysis computerized system.[10]

Woodward Triplet

In general this code is a surface panel method employing a special type of surface singularity that is referred to as a triplet because it is a combination of a source and a doublet.

For supersonic flows, surface panel methods must not be allowed to produce any spurious waves that can and usually do destroy the solution. In applications of the USSAERO code,[11] these undesirable waves were first encountered as propagating internally for boattailed configurations; in the USSAERO code, the body representation consists of surface source panels only. A basic triplet singularity was formulated for bodies of revolution and successfully alleviated the in-

GENERAL SLENDER BODY THEORY	• LINEAR THEORY • LINE SOURCES AND SINKS FOR AXIAL FLOW COMPONENT • THEODORSEN TRANSFORMATION FOR CROSS FLOW COMPONENTS • SUBSONIC/SUPERSONIC SPEEDS
WOODWARD TRIPLET	• CONSTANT SOURCE, CONSTANT LINEAR VORTICITY SURFACE SINGULARITY • VELOCITY BOUNDARY CONDITION • SUPERSONIC SPEEDS
PAN AIR	• LINEAR SOURCE, QUADRATIC DOUBLET SINGULARITY • INTERIOR POTENTIAL BOUNDARY CONDITIONS • SUBSONIC/SUPERSONIC SPEEDS
MODIFIED LINEAR THEORY	• MODIFIED LINEAR THEORY • SURFACE DISTRIBUTIONS OF SOURCES AND SINKS • SUPERSONIC SPEEDS
SHOCK-FITTING FINITE DIFFERENCE	• EXACT INVISCID EQUATIONS • CONTINUOUS SURFACE SHAPE • MARCHING TECHNIQUE • SURFACE AND FIELD DATA

Fig. 1 Program characteristics.

terior wave propagation problem. The basic triplet was then extended to bodies of arbitrary cross sections (requiring the development of an axial, circumferential and radial triplet)[2] and has been incorporated into the USSAERO code. The addition of higher-order terms to the Prandtl-Glauert equation can provide improved estimates of surface pressures. However, these higher-order terms result in nonlinear partial-differential equations that are not amenable to panel method solutions. As an option, an approximate local Mach number correction is included in the USSAERO modifications. In this method a solution based on the freestream Mach number is determined and a second solution is obtained based on the average of the freestream and the local Prandtl-Glauert factors. This solution is a first approximation of the nonlinear pressures, forces, and moments. It is this modified version of the USSAERO code, both with and without local Mach number correction, that was used in making this assessment.

PAN AIR Pilot Code

This code was developed by Boeing Aircraft Corporation under contract to several U.S. Government agencies as a technology demonstrator for a subsequent production code.

This method of aerodynamic analysis uses a surface paneling approach in which linearly varying sources and quadratically varying doublets are distributed on quadrilateral panels; this is sometimes referred to as a higher-order surface paneling method.[12,13] For these higher-order doublet panels, the doublet strength is forced to be continuous across all panel edges and all adjacent panels are assured to have contiguous edges. This eliminates the generation of any spurious line vortex behavior which can produce disastrous numerical problems for supersonic flows. This is another approach for avoiding undesirable wave propagation that was addressed by Woodward with the triplet singularity.

Although most aerodynamic analysis codes are formulated about a specific set of fixed boundary conditions, the PAN AIR pilot code provides the capability of specifying boundary conditions directly in terms of the perturbation potential or indirectly in terms of mass flux or velocity components. In Ref. 14 the velocity boundary conditions were shown to provide better supersonic estimates for an ogive-cylinder than the mass flux boundary conditions; thus, all of the PAN AIR pilot code results for this study were calculated with velocity boundary conditions.

Modified Linear Theory

This computer code is being developed by Vought Corporation to fill a gap in the zero-lift wave drag calculation capability. It is well known that the widely used and very general far-field wave drag calculation procedure[15] does not produce correct results when the configuration surface slope angle exceeds the Mach angle. This situation occurs frequently at some location on most aircraft at high Mach numbers and on fighter-type aircraft having drooped noses or high visibility canopies at moderate Mach numbers.

Near-field linearized theory surface panel methods are subject to the same limitations as far-field methods; however, certain modifications to linear theory have overcome most of these limitations. The method was first successfully applied to the calculation of thickness drag of planar surfaces (wings and tails).[4] In this formulation two modifications have been made to what is basically a linearized theory surface-source analysis method. The first modification entails the use of local or perturbed values of Mach number for calculating the perturbation velocities; the effect of using local Mach number shows up in evaluating almost every term of the aerodynamic influence coefficient matrix and in determining the region of influence. The region of influence is no longer determined by the Mach forecone but is determined by tracing the forward running characteristics from the control point where boundary conditions are to be satisfied. This use of local Mach number requires an iterative solution technique. The second modification is the use of exact rather than linearized boundary conditions; this is accomplished by requiring that the velocity vector (freestream plus perturbation) normal to the surface be zero on the surface. The pressure coefficient is computed using the exact isentropic relationship.

Shock-Fitting Finite-Difference Method

The general numerical scheme developed by Moretti[16] for solving the Euler equations for supersonic flow about complex configurations has been extended and implemented for digital computations.[5,17] A finite-difference marching scheme of second-order accuracy is used to compute the complete flowfield between the body and the bow shock. In this method the flow equations are recast to give the derivatives of the flow variables in the marching direction (along the body axis) in terms of the quantities and their derivatives in a plane perpendicular to the marching direction. Starting from a given data plane, the derivatives are integrated a single step forward (along the body axis) using a MacCormack two-level predictor-corrector finite-difference scheme to obtain a new data plane. This is repeated until the end of the body is reached. Before each step, the maximum step-size is computed in order to satisfy the Courant-Friedricks-Levy stability criterion for stability. A conformal mapping of the region between the bow shock and the body into a rectangular region produces the computational grid. When embedded shock waves occur, the mesh is adjusted so that the mesh lines coincide with those shocks and the Rankine-Hugonoit relations are satisfied explicitly across each of the shock waves.

To avoid generating spurious shocks, the configuration is defined as a smooth surface (continuous first derivatives) by analytic functions (generally conic sections) through relatively few prescribed points. The present implementation of the technique is somewhat restricted in the configurations it can handle. First, the configuration cross section must be single valued in polar coordinates. In addition, modification of the configuration nose may be required by either of the two methods currently in use for generating the starting data plane. One method is to begin with a circular cone whose half-angle must be greater than the maximum angle of attack. The other choice is to begin with a blunt nose, that requires the use of a separate program to get past the subsonic region and provide a supersonic starting plane.

Other than geometry, the only inherent limitation in finite-difference marching techniques is that the velocity component in the marching direction must always be supersonic. However, improvements to the method of Ref. 5 (reported in Ref. 17) have relaxed this restriction to allow subsonic velocities in the marching direction as long as the total velocity is supersonic.

Discussion

Input Geometry

Except for the finite-difference code, the mathematical model of the basic geometry was defined using the format of Ref. 18. This mathematical model was input directly along with auxiliary data as needed to execute the Woodward triplet and the general slender body codes. For the modified linear theory and the PAN AIR codes, the geometry input was generated from this same mathematical model by intermediate geometry reformatting computer codes.

The geometry input for the finite-difference code was entirely analytic and was obtained by describing the configuration in terms of lines and simple analytic curves.

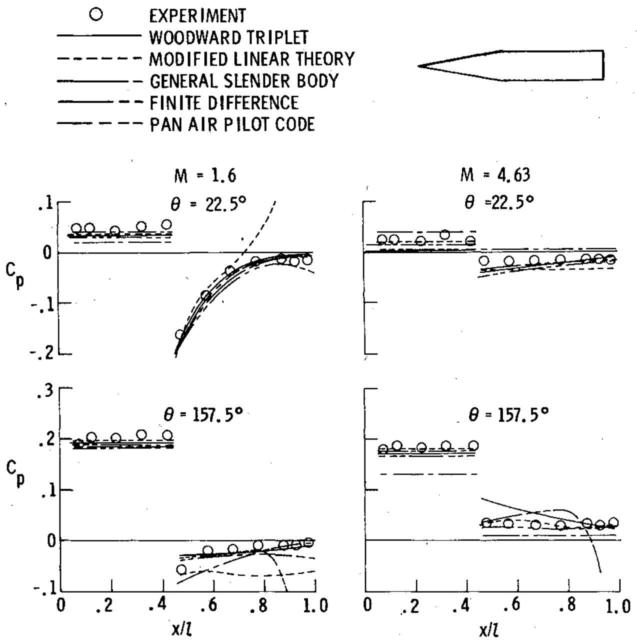

Fig. 2 Cone cylinder, $\alpha = 8$ deg.

Comparison of Prediction Methods With Experiment

In order to provide comparisons between the five prediction methods and experimental data, surface pressures for several different body shapes were calculated for a range of Mach numbers and angles of attack. Results for two of those body shapes, a 9.5 deg cone cylinder and an analytically-developed forebody (which resembles the cockpit region of a fighter aircraft) were chosen for presentation in this paper. These two body shapes were the simplest and the most complex of those for which experimental data are presented in Refs. 6-9.

The data shown in Fig. 2 for cone cylinder at $\alpha = 8$ deg is typical of the results historically obtained for such a body except for the pressure excursion on the cylinder at $M = 1.6$ by the modified linear theory. This excursion appears to be a correctable problem associated with the tracing of characteristics and was observed in other calculations. On the lower surface, the modified linear theory overpredicts the low Mach number expansion on the cylindrical afterbody and the PAN AIR pilot code produces a pressure excursion at both the low and the high Mach numbers. In general, results for the cone cylinder showed little difference between the prediction methods. Differences between experimental and computed pressures tend to increase with increasing Mach number and angle of attack.

The analytically-developed forebody results shown in Fig. 3 are typical and provide a better insight into the validity of the five codes considered. Only two of the five methods (modified linear theory and shock-fitting finite-difference) produce adequate results in regions where nonlinear effects become

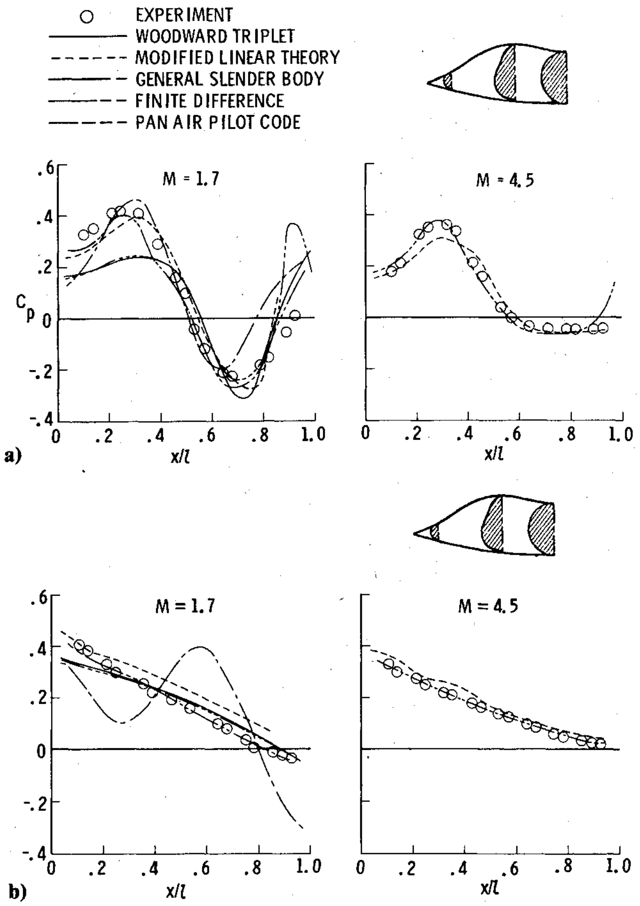

Fig. 3 Analytically developed forebody, $\alpha = 5$ deg; a) $\theta = 0$ deg, b) $\theta = 180$ deg.

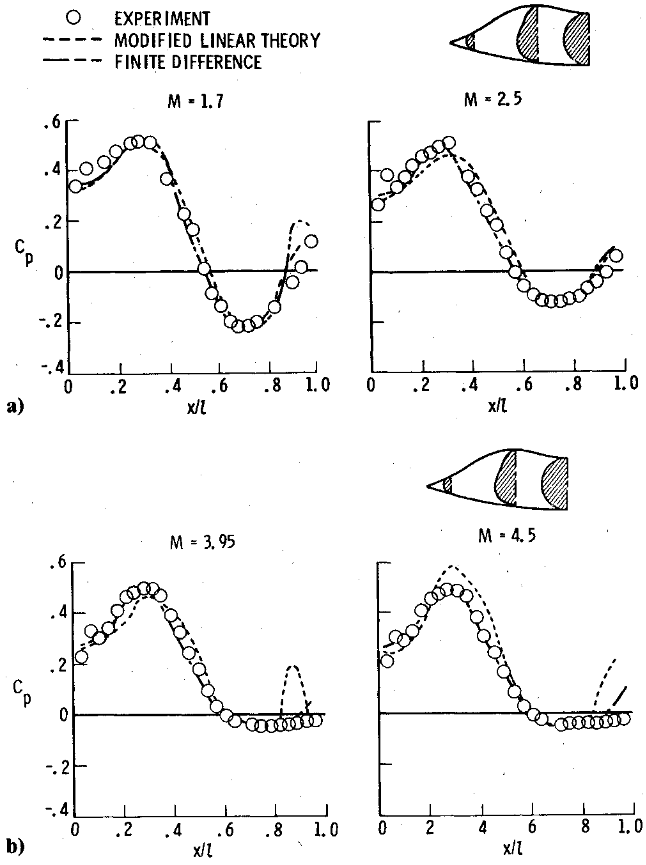

Fig. 4 Effect of Mach number, analytically developed forebody, $\alpha = 0$ deg, $\theta = 0$ deg; a) $M = 1.7$ and 2.5; b) $M = 3.95$ and 4.5.

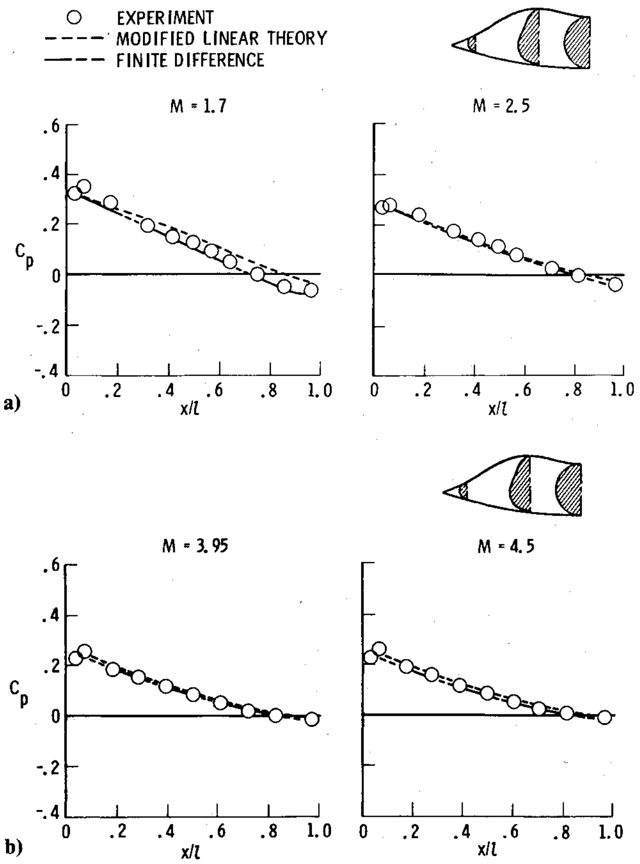

Fig. 5 Effect of Mach number, analytically developed forebody, $\alpha = 0$ deg, $\theta = 180$ deg; a) $M = 1.7$ and 2.5, b) $M = 3.95$ and 4.5.

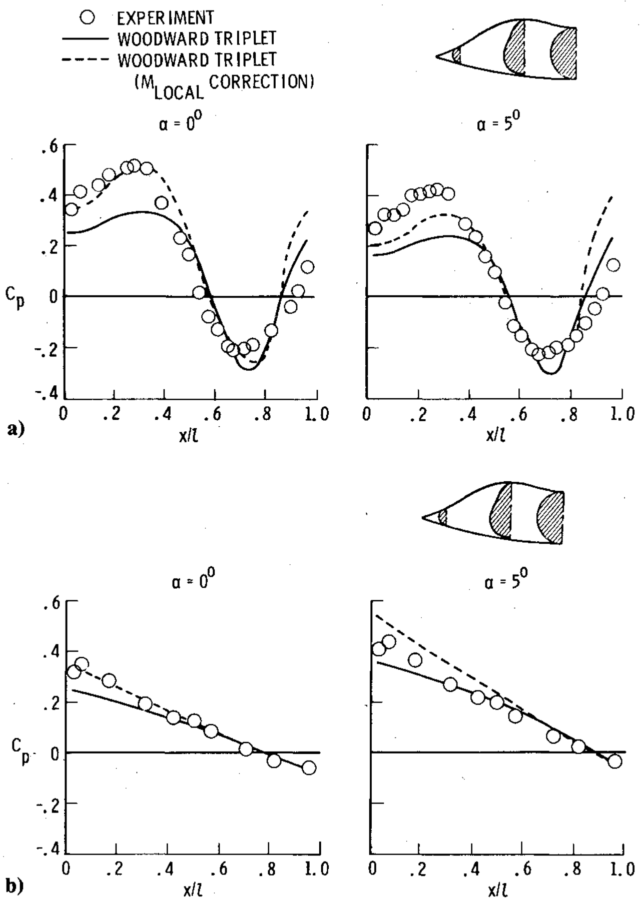

Fig. 6 Local Mach number correction, analytically developed forebody, $M = 1.7$; a) $\theta = 0$ deg; b) $\theta = 180$ deg.

important; in fact, due to Mach angle limitations, the linear theory panel methods could not produce any results for some flow situations. On the upper surface in the region of the canopy where flow perturbations are not small both linearized panel methods similarly underpredict the compression pressures; in this region the nonlinear effects must be considered. The lower surface pressures from the general slender body code appear, for some reason, to be following opposite trends to those of the upper surface. These poor results are probably due to the fact that this body is not a slender body. Because of the Mach angle limitations of the linearized panel methods (Woodward triplet and PAN AIR pilot code) and the extremely poor results of the general slender body theory, only the modified linear theory and the shock-fitting finite-difference methods produce acceptable results for the analytic forebody.

Since the modified linear theory is basically a surface panel-type code designed to approximate the exact solution with a simple and easily-generated input requirement and the finite-difference code is designed to produce an exact solution from a complex but more difficult to generate input, further analytic forebody results are presented in Figs. 4 and 5 to indicate the solution differences one might expect between these two methods as Mach number increases. These results show excellent agreement between the finite-difference method and experimental data for all of the Mach number angle-of-attack combinations. The modified linear theory pressures appear to include the dominant nonlinearities; however, on the upper surface (Fig. 4), the agreement between theory and experiment for the modified linear theory decreases with increasing Mach number while the opposite trend occurs on the lower surface (Fig. 5).

In an effort to strike a compromise between computing complexity of the modified linear theory and the simplicity of the Woodward triplet, a simple local Mach number correction has been made to the basic triplet formulation. Although still limited by the Mach angle restriction, the local Mach number modification in general offers an improvement over linearized theory results. In Fig. 6 the results for the analytic forebody show the correction produces excellent results for $\alpha = 0$ deg but not as good at $\alpha = 5$ deg. Similar results were also obtained for the cone cylinder at the higher Mach numbers.

Computer Solution Time

In many aerodynamic analysis computer code applications, solution time is just as important a factor as generality, range of applicability, accuracy, etc. Thus, a discussion of methods should include some mention of solution times; however, it is important to remember that the five methods presented in this paper are at various states of development and the times given here represent the present situation.

For each of the five computer codes, the central processor time (CPU seconds) for a CDC Cyber 175 is shown in Fig. 7 for the range of body panels or grid points used in making the calculations shown in the previous section.

Surprisingly, the shock-fitting finite-difference code is competitive with the other codes with respect to time. This may be due in part to built-in optimization procedures for determining step size. Although fewer panels are needed to provide realistic results with the modified linear theory code, the times are rather high when compared with those for the Woodward triplet or the general slender body codes with larger numbers of panels. The PAN AIR pilot code solution times (without panel blocking in Fig. 7) were the longest of any obtained. For a body alone with this code one would hope for times of the order of those for the Woodward triplet or the general slender body codes. Considerable effort has been expended to reduce computing time for this code since the original data were run. PAN AIR computing times for an unreleased version of the code which employs "panel

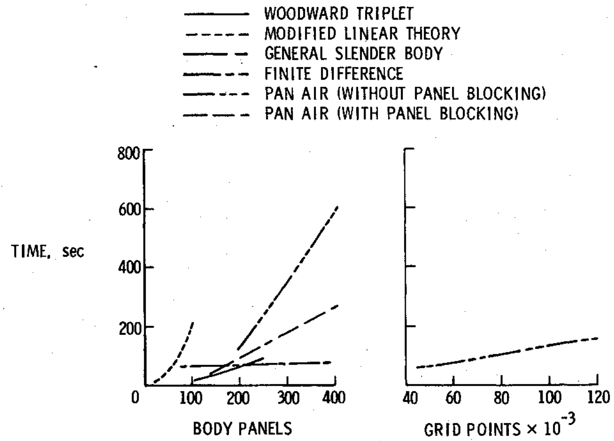

Fig. 7 CYBER 175 computer times.

blocking" are also presented in Fig. 7. When the far-field influence is adequate, the panel blocking technique forms groups of panels into a single panel having a single influence coefficient. PAN AIR computing times with panel blocking are now competitive with the other codes. It is hoped that further optimization of the modified linear theory code will provide significant computing time reductions.

Future Considerations

As previously noted, each of the five codes being examined in this paper is at some stage of its development. In this section, the near term plans for each code are presented.

General Slender Body

Incorporation of the general slender body code into a supersonic design and analysis computerized system will be completed. In order to handle fuselage inlets, jump conditions for surface discontinuities are also being included.

Woodward Triplet

The triplet concept appears to work rather well except for arbitrary bodies (particularly ellipses) with the major axis vertical. Care must be exercised when analyzing such configurations because the solution appears to become unstable. A generalized triplet singularity applicable to both bodies and wings is being developed and will be incorporated in the USSAERO code. An alternate set of more general panel corner point inputs will also be included.

PAN AIR Pilot Code

The PAN AIR pilot code is being translated into a production code using structured programming concepts. Care must be taken to assure that production code execution time is no longer than pilot code time, preferably shorter.

Modified Linear Theory

The code is being extended to include wing-body combinations. There is obviously a need to improve execution time.

Shock-Fitting Finite-Difference

An effort is being made to develop and incorporate the λ integration scheme and an advanced mapping technique. The λ scheme has the properties of a method of characteristic-type solution but the simplicity of a typical finite-difference method. The mapping method has removed geometry limitations to include highly cambered arrow wings. A preliminary version of a new program incorporating these advances, but limited to very specific configurations, has produced good results.

Concluding Remarks

In making this assessment of five supersonic flow analysis computer codes, it became clear that the need still exists for a code that can account for the predominant nonlinear effects, operate from an easily generated and manipulated geometry data set (surface panel type), and execute in a reasonable time.

The major findings of this study can be summarized as follows: Only two codes, the modified linear theory and shock-fitting finite-difference, provided good results in all cases considered. The main shortcoming of these two codes is the excessive execution time of the modified linear theory code and the unwieldy geometry requirements of the shock-fitting finite-difference code. In analyzing a typical fuselage-type forebody, the linearized theory methods are severely limited in their applicability; they produce incorrect solutions in regions where nonlinearities are significant and no solutions in regions where the surface slope exceeds the Mach angle. It is not uncommon for these types of regions to exist in several places on aircraft fuselages.

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